

PROBLEM SET 2

Due: October 2, 2003.

Problem 1

Halzen and Martin, exercise 14.6

Problem 2: Higgs Mechanism

Starting from

$$\mathcal{L} = \frac{1}{2}(\mathcal{D}_\mu\phi)^*\mathcal{D}^\mu\phi + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},$$

where $\mathcal{D}_\mu \equiv \partial_\mu - ieA_\mu$ is the covariant derivative, $\phi = \phi_1 + i\phi_2$ is a complex scalar field, λ and μ are constants ($\lambda^2 > 0$ and $\mu^2 > 0$), and selecting new fields η, ξ ,

$$\eta = \phi_1 - \mu/\lambda, \quad \xi = \phi_2,$$

derive

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2 + \frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{e\mu}{\lambda}\right)^2 A_\mu A^\mu \\ & + \left(\frac{\mu e}{\lambda}\right)(\partial_\mu\xi)A^\mu - e[\eta(\partial_\mu\xi) - \xi(\partial_\mu\eta)]A^\mu + \frac{\mu}{\lambda}e^2\eta(A_\mu A^\mu) + \frac{1}{2}e^2(\xi^2 + \eta^2)(A_\mu A^\mu) \\ & - \lambda\mu(\eta^3 + \eta\xi^2) - \frac{1}{4}\lambda^2(\eta^4 + 2\eta^2\xi^2 + \xi^4) + \left(\frac{\mu^2}{2\lambda}\right). \end{aligned}$$

Interpret this resulting Lagrangian in terms of particles and draw the interaction vertices. Which of these interactions survive the Higgs mechanism?

Problem 3

Halzen and Martin, exercise 12.2