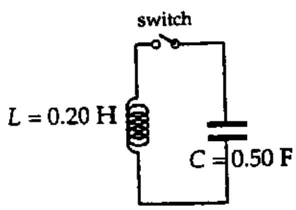
Consider this circuit, which has $C = 0.50$ F and $L = 0.20$ H. With the switch open (as drawn here), the capacitor is charged up, using a 2-volt battery. The battery is removed. Then, at time $t = 0$, the switch is closed.



(a) Immediately before the switch is closed, what's the charge on the capacitor? Call your answer "Q_0."

(b) Set up an equation that could be solved for Q, the charge on the capacitor, as a function of time. Do not solve this equation, unless you want to.

(c) Sketch a rough graph of i vs. t (after $t = 0$), where i denotes the current through the inductor. This graph need not be numerically accurate. Just worry about the general shape.

(d) When the charge on the capacitor is $Q_0/2$, what current is flowing through the inductor? Hint: You need not solve the differential equation from part (b). Instead, think about energy.

(e) The period of oscillation of the LC circuit is $T = 2\pi\sqrt{LC}$. (I figured this out by solving the differential equation from part b.) What is the earliest time, after $t = 0$, when the capacitor has no charge on it?

(f) Explain, intuitively, why the period of oscillation increases if we raise the inductance, *or* if we raise the capacitance. It might help you to consider an analogy with a mass on a spring.

a) $Q_0 = CV_0$ (steady state $\Rightarrow$ inductor doesn't matter)

b) Use the Loop Rule $\quad \mathcal{E}_{L_1} + V_0 = 0$

$$-L\frac{di}{dt} + \frac{Q}{C} = 0$$

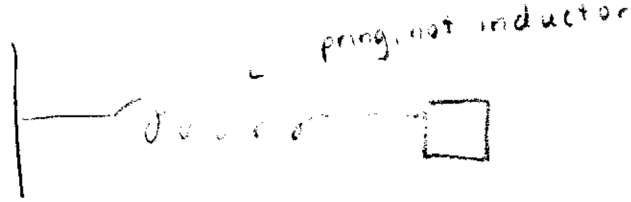
But $i = -\dfrac{dQ}{dt}$ (leaving cap) $\Rightarrow \boxed{L\dfrac{d^2Q}{dt^2} + \dfrac{Q}{C} = 0}$

This is a DE for Q

c) $$L\frac{d^2Q}{dt^2} = -\frac{1}{C}Q$$

This is the same equation as for a block on a spring!

Remember



$$\boxed{F = ma = m\frac{d^2x}{dt^2}} = -kx$$

$$L\,\frac{d^2Q}{dt^2} = -\frac{1}{C}Q$$

same math!
Same solution!

$Q \leftrightarrow x$
$L \leftrightarrow m$
$\frac{1}{C} \leftrightarrow k$

Solution: Sinusoid $\quad Q = Q_0 \sin(\omega t + \phi)$

$$I = \frac{dQ}{dt} = Q_0 \omega \cos(\omega t + \phi)$$

Shape looks like:

d) Use energy conservation

$$U_C = \frac{1}{2}\frac{Q^2}{C}, \quad U_L = \frac{1}{2} L I^2$$

Note mechanical analogy! $\quad \frac{1}{2}kx^2$ (like spring PE) $\qquad \frac{1}{2}mv^2$ (like KE)

when $Q = \frac{Q_0}{2}$ $\qquad$ initially when $Q = Q_0$ $\qquad$ we are trying to find this

$$U_C + U_L)_f = U_C + U_L)_i$$

$$U_{C_i} = \frac{1}{2}\frac{Q_0^2}{C}, \quad U_{L_i} = 0; \quad U_{C_f} = \frac{1}{2}\left(\frac{Q_0}{2}\right)^2\frac{1}{C}, \quad U_{L_f} = \frac{1}{2}L I_f^2$$

$$\frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\left(\frac{Q_0}{2}\right)^2\frac{1}{C} + \frac{1}{2}L I_f^2$$

solve for I_f : $\quad I_f^2 = \frac{1}{L}\left[\frac{3Q_0^2}{4C}\right]$

When $Q = Q_0$ $\quad I_f = \sqrt{\frac{3Q_0^2}{4CL}} = \underline{\underline{2.7\ A}}$

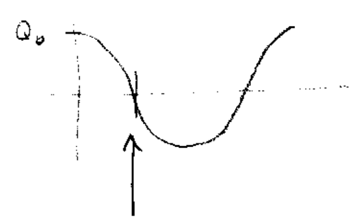
e) Our DE is $\quad \frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$

The sinusoid solution is for $\omega = \sqrt{\frac{1}{LC}}$

(just as for springs it was $\omega = \sqrt{\frac{k}{m}}$)

So $\quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$

The charge swings from Q_0 to 0 to $-Q_0$ to 0 to Q_0 in one period



So the first time it has $Q = 0$ is $\frac{1}{4}$ period

$\Rightarrow t = \frac{1}{4}T = \frac{1}{4}2\pi\sqrt{LC} = \underline{\underline{0.5\ s}}$

f) Mathematically: $T \propto \sqrt{LC}$ so it increases if $L\uparrow$, $C\uparrow$

Intuitively: → L is like inertial mass

mechanical analogy

increasing it makes the circuit respond more slowly, so it sloshes back and forth more slowly

→ $1/C$ is like the spring constant

Increasing C is like decreasing the spring constant, which makes for a (weaker spring) slower oscillation.

(or: increasing C means more charge can get stuffed on, which means it takes longer to respond)