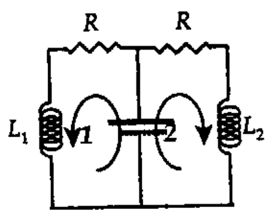


A variable-voltage power supply is connected to two inductors, as drawn here. I've used the standard "battery" icon to indicate the power supply. A small picture of a solenoid always denotes an inductor, whether or not the inductor happens to be a solenoid.

The left inductor has inductance $L_1 = 0.4$ henrys. The right inductor has inductance $L_2 = 0.6$ henrys. The power supply is currently set to $V_0 = 10$ volts. Both inductors are in series with resistors of resistance $R = 2\ \Omega$.

(a) Assuming the power supply has been turned on for a long time, what current flows through the left inductor? The right inductor?

(b) Suddenly, at a moment I'll call $t = 0$, the power supply is abruptly turned down to 0 volts. But current can still flow through its inner workings. Using qualitative reasoning, without doing any calculations, tell me which inductor has more current flowing through it at time $t = 0.1$ s. Explain your reasoning in detail.

(c) A long time after the power supply is set to 0 volts, what current flows through the right inductor? The left inductor? Why?

(d) Now write a differential equation that could be solved for i_1, the current through the left inductor, at arbitrary time t after the power supply is turned to zero. You need not solve the equation, unless you want to. But try to set it up.

(e) Sketch a rough graph of i_1 vs. time and i_2 vs. time. Let $t = 0$ be the moment the power supply was turned to 0 volts. These graphs need not be numerically accurate.

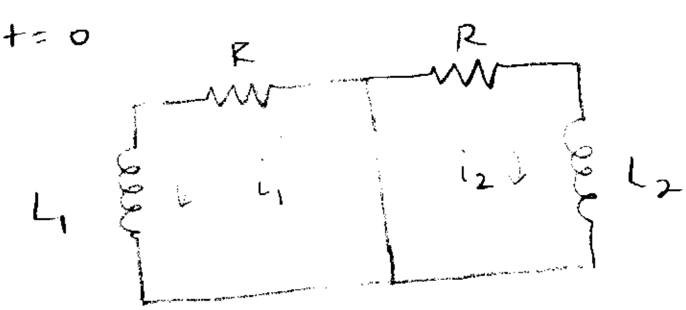
a) After a long time (in the steady state) there is no current change, so no induced emf

$$\mathcal{E}_L = -L\,di/dt = 0 \Rightarrow \boxed{\text{Inductor has no effect}}$$

The current is just given by Ohm's Law

$$i_1 = i_2 = \frac{V_0}{R} = \frac{10\text{ V}}{2\ \Omega} = \underline{5\text{ A}}$$

b) At $t=0$



The inductors resist charge in current.
The bigger L, the bigger the opposition, and the longer it will take current to settle down

$\Rightarrow$ since $L_2 > L_1$, $\underline{\underline{L_2}}$ will have a greater current at any given time

c) At very long times, the current has died away completely $\Rightarrow$ $\underline{\underline{i = 0}}$ through both

d) Choose current direction and apply the loop rule

$$-i_1 R + \mathcal{E}_{L_1} = 0$$

$$-L \frac{di_1}{dt} = \mathcal{E}_{L_1} \Rightarrow i_1 R + L_1 \frac{di_1}{dt} = 0$$

A D.E. we can solve for the current

Let's solve it in order to make part e easier

Separate variables: $\dfrac{di_1}{i_1} = -\dfrac{R}{L_1} dt$

Integrate $\displaystyle\int_{i_{10}}^{i_1} \frac{di_1'}{i_1'} = -\frac{R}{L_1}\int_0^t dt'$

initial current $\rightarrow i_{10}$

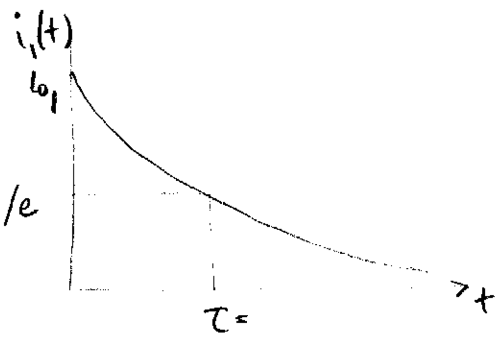
$$\ln i_1 - \ln i_{10} = -\frac{R}{L_1} t$$

$$\ln\left(\frac{i_1}{i_{10}}\right) = -\frac{R}{L_1} t$$

$$\frac{i_1}{i_{10}} = e^{-(R/L_1)t}$$

$$\boxed{i_1(t) = i_{10} e^{-(R/L_1)t}}$$

This is the current as a function of time after the switch is closed (similar eqn for $i_2(t)$)

e) 

$i_1(t) = i_{01}$ at $t = 0$

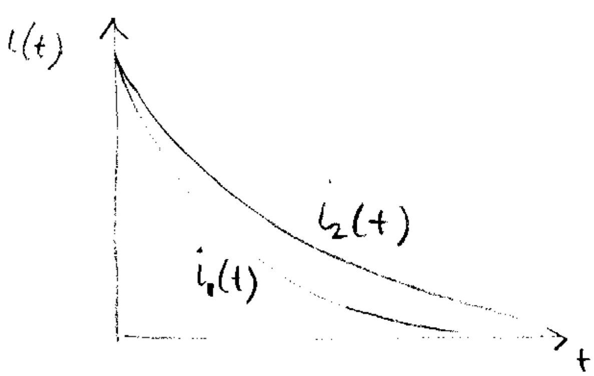
$i_1 \to 0$ as $t \to \infty$

Time constant $\tau = L/R$

$$i = i_0 e^{-t/\tau}$$

bigger $L \Rightarrow$ bigger τ

So compared to $i_1(t)$, $i_2(t)$ should be more spread out (takes longer to respond)



Mechanical analogy: $L \rightarrow m$

$R \rightarrow$ drag

$$m\frac{d^2x}{dt^2} = -bv = -b\frac{dx}{dt}$$

Mass with initial velocity under influence of drag force

Solution

$$v = v_0 e^{-t(b/m)}$$

slows to a stop