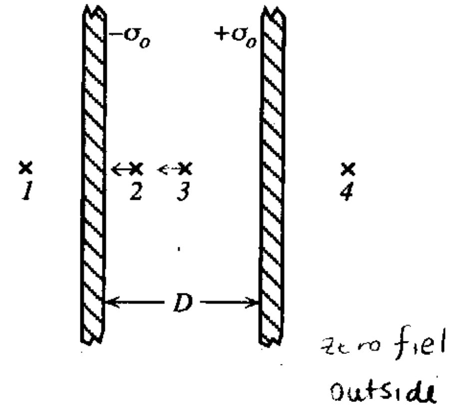
II. Parallel plates and capacitance

Two very large thin conducting plates are a distance D apart. The surface area of the face of each plate is A_o. A side view of a small portion near the center of the plates is shown.



A. The inner surface of one plate has a uniform charge density of $+\sigma_o$; the other, $-\sigma_o$. The charge density on the outer surface of each plate is zero.

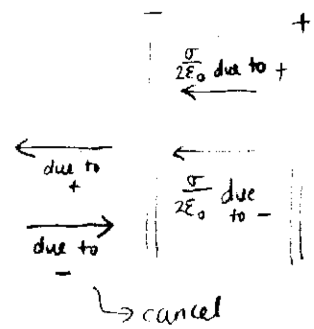
1. At each labeled point, draw vectors to represent the electric field at that point due to each charged plate.

2. Write expressions for the following quantities in terms of the given variables:

   - the electric field at points *1*, *2*, *3*, and *4*

$$\vec{E}_1 = \vec{E}_4 = 0$$

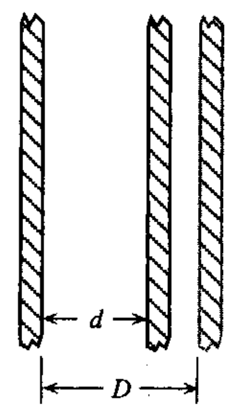
$$\vec{E}_2 = -\frac{\sigma}{\varepsilon_0}\hat{\imath}, \quad \vec{E}_3 = -\frac{\sigma}{\varepsilon_0}\hat{\imath}$$



   - the potential difference between the plates

$$\Delta V = \int \vec{E}\cdot d\vec{x} = E\int_0^D dx = ED$$

3. The right plate is moved to the left as shown. Both plates are kept <u>insulated</u>. Describe how each of the following quantities will change (if at all). Explain.



   - the charge density on each plate

   Doesn't change – charge has nowhere to go!

   - the electric field both outside and between the plates

   Doesn't change – depends on σ only

   - the potential difference between the plates

$$\Delta V = \int \vec{E}\cdot d\vec{x} = Ed$$

   decreases since $d < D$

4. Write expressions for the following quantities in terms of σ_o and d (the new distance between the plates).

   • the magnitude of the electric field between the plates

   $$E = \frac{\sigma_0}{\varepsilon_0} \quad \text{as before}$$

   • the potential difference between the plates

   $$\Delta V = Ed = \frac{\sigma_0}{\varepsilon_0} d$$

5. Find $\frac{Q}{\Delta V}$ (the ratio of the net charge on one plate to the potential difference between the plates).

   $$Q = \sigma_0 A$$

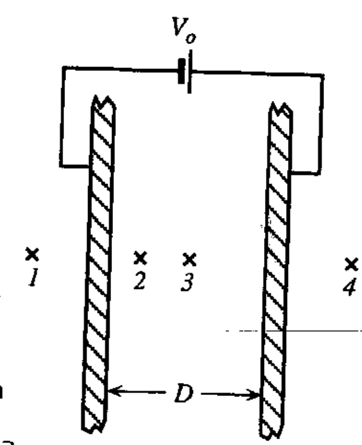
   $$\frac{Q}{\Delta V} = \frac{\sigma_0 A}{\frac{\sigma_0}{\varepsilon_0} d} = \frac{\varepsilon_0 A}{d}$$

   How, if at all, would this ratio change if the charge densities on the plates were $+2\sigma_o$ and $-2\sigma_o$?

   Would not change – it does not depend on σ

➪ Check your results for part A with a tutorial instructor before you continue.

B. Suppose the plates are discharged, then held a distance D apart and connected to a battery. (Ignore the fringing fields near the plate edges.)



1. Write expressions for the following quantities in terms of the given variables. Explain your reasoning in each case.

   • the potential difference ΔV between the plates

   $\Delta V = V_0$ That's what a battery does

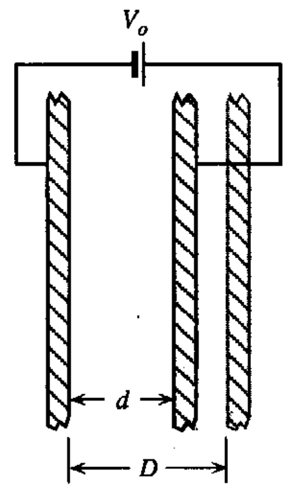
   • the electric field at points *1*, *2*, *3*, and *4*

   $$E_1 = E_4 = 0, \quad \vec{E}_2 = -\frac{V_0}{D}\hat{i} = \vec{E}_3$$

   • the charge density on each plate

   $$|\sigma_0| = \varepsilon_0 E = \frac{\varepsilon_0 V_0}{D}$$

2. The right plate is moved to the left. Describe how each of the following quantities changes (if at all). Explain.



   - the potential difference ΔV between the plates

     Same $\Delta V = V_0$

   - the electric field both outside and between the plates

     Outside $E = 0 \Rightarrow$ same

     Inside $E = \frac{V_0}{d} \Rightarrow$ bigger since $d \downarrow$

   - the charge density on each plate

     $|\sigma| = \frac{\varepsilon_0 V_0}{d} \Rightarrow$ bigger, since $d \downarrow$

3. Write expressions for the following quantities in terms of V_o and d (the new distance between the plates).

   - the magnitude of the electric field between the plates

     $E = \frac{V_0}{d}$

   - the charge density on each plate

     $\sigma = \frac{\varepsilon_0 V_0}{d}$

4. Find $\frac{Q}{\Delta V}$ (the ratio of the net charge on one plate to the potential difference between the plates).

   $Q = \sigma A$

   $\frac{Q}{\Delta V} = \frac{\varepsilon_0 \left(\frac{V_0}{d}\right) A}{V_0} = \frac{\varepsilon_0 A}{d}$ Same as part A!

   How, if at all, would this ratio change if the voltage of the battery was $2V_o$?

   Same; this answer is independent of V_0

⇨ Check your results for part B with a tutorial instructor before you continue.

C. Compare the ratio $\frac{Q}{\Delta V}$ that you calculated for two insulated plates (part A) to the same ratio for two plates connected to a battery (part B).

1. Does the ratio $\frac{Q}{\Delta V}$ depend on whether or not the plates are connected to a battery?

No

2. Does the ratio $\frac{Q}{\Delta V}$ depend on the distance between the plates?

Yes

The potential difference ΔV between two isolated conductors depends on their net charges and their physical arrangement. If the conductors have charge $+Q$ and $-Q$, the ratio $\frac{Q}{\Delta V}$ is called the *capacitance (C)* of the particular arrangement of conductors.

D. For the following cases, state whether each of the quantities q, σ, E, ΔV, and C changes or remains fixed:

1. two insulated conducting plates are moved farther apart (no battery)

q same
σ same
E same
ΔV increase
$C = \frac{Q}{\Delta V}$ decrease

2. two conducting plates connected to a battery are moved farther apart

q decrease
σ decrease
E decrease
ΔV same
$C = \frac{Q}{\Delta V}$ decrease