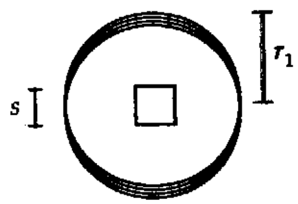
END-ON VIEW, looking down the solenoid tube. It extends distance l into the page.



Consider a standard cylindrical solenoid of length l and radius r_1 consisting of N coils. A small metal square of wire, of side length s and resistance R, is placed inside the solenoid. The plane of the square is parallel to the plane of a solenoid coil.

Initially, no current flows through the solenoid. But starting at time $t = 0$, a variable power supply steadily increases the current through the solenoid from 0 to I_f, over time T.

In this problem, you may use answers previously derived in this chapter. You need not rederive them from scratch.

(a) In terms of the given physical quantities, find dI/dt, the rate of change of current *in the solenoid*.

(b) While the current is increasing, what is the induced EMF *in the solenoid*?

(c) Let "M" denote the "mutual inductance" between the solenoid and the square. By definition, it's the proportionality constant relating the rate of change of current *through the solenoid* to the induced EMF (voltage) ***in the square:***

$$\mathcal{E}_{\text{induced in square}} = -M\frac{dI_{\text{solenoid}}}{dt}.$$

Find the mutual inductance between the solenoid and the square.

(d) Explain in simple language the physical difference between self-inductance (L) and mutual inductance (M).

(e) Between time $t = 0$ and $t = T$, how much total heat dissipates in the wire square?

a) $$\frac{dI}{dt} = \frac{I_f - 0}{T - 0} = \frac{I_f}{T}$$

b) $$\mathcal{E}_{\substack{ind \\ sol}} = -\underbrace{L}_{\text{found in previous problem}}\frac{dI}{dt} = \mu_0 \frac{N^2}{l}\pi r_1^2 \; \frac{I_f}{T}$$

c) $$\mathcal{E}_{\substack{ind \\ square}} = -M\frac{dI_{sol}}{dt}$$

$$\Phi_{B\,square} = (\text{stuff})\, I_{sol} \qquad \circledast$$

$$\frac{d\Phi_{B\,square}}{dt} = (\text{stuff})\frac{dI_{sol}}{dt}$$

$$\mathcal{E}_{\substack{ind\\square}} = -\frac{d\Phi_{Bsq}}{dt} = -(\text{stuff})\frac{dI_{sol}}{dt}$$

The "stuff" in ⊛ is M

$$\Phi_{Bsq} = M\, I_{sol}$$
$$= B \cdot A = \mu_0 \frac{N}{\ell} I_{sol}\, s^2$$

$$\Rightarrow \boxed{M = \mu_0 \frac{N}{\ell} s^2}$$

d) L : sensitivity to change in <u>own</u> current

M : sensitivity to change in <u>another</u> current

e) $P = \dfrac{\mathcal{E}_{ind}^{2}}{R}$

$$\mathcal{E}_{ind\,sq} = \frac{\mu_0 N}{\ell} s^2 \frac{I_f}{T}$$

$$P = \frac{\mu_0^2 N^2}{\ell^2} s^4 \frac{I_f^2}{RT^2} \quad (\text{constant})$$

$$\text{Heat} = P \cdot T = \frac{\mu_0^2 N^2}{\ell^2} s^4 \frac{I_f^2}{RT}$$