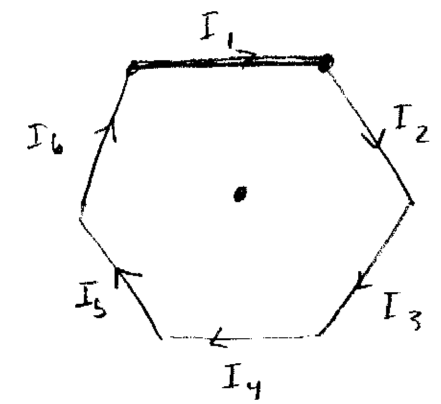
Take six segments of wire, each of length s and resistance R, and connect them into a nexagon. A tiny battery makes current flow clockwise around this circuit, by supplying voltage $\mathcal{E}_0$. The hexagon "lives" in the plane of this page.

(a) What's the magnetic field at the center of the hexagon? If you get stuck in the math, that's OK; but try to set everything up.

(b) A particle of charge Q is at the center of the hexagon, moving with speed v_0 into the page. What is the magnitude and direction of the magnetic force on that charge? Qualitatively describe the particle's trajectory in the long run.

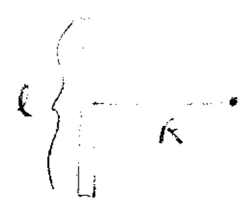
(c) Now a particle of charge Q is at the center of the hexagon, moving rightward at speed v_0. Initially, what is the magnitude and direction of the force on the particle? For extra credit, qualitatively describe the particle's trajectory.

a)



Step 1 : break into segments, label them

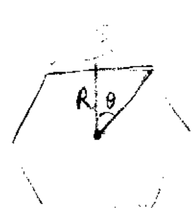
Step 2 : Here we can use our known result, from problem 36



$$B(R) = \frac{\mu_0 I}{2\pi R} \frac{\ell}{(\ell^2 + 4R^2)^{1/2}}$$

Corresponding quantities for octagon side : $\ell \to s$

$\tan\theta = \frac{s/2}{R}$



$R = \frac{s}{2\tan\theta}$ where $\theta = \frac{180^\circ}{6}$

$R = \frac{s}{2\tan 30^\circ} = \frac{s\sqrt{3}}{2}$

$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

So the field from one side is

$$B = \frac{\mu_0 I}{\cancel{2}\pi \frac{s\sqrt{3}}{\cancel{2}}} \frac{s}{\left(s^2 + 4 s^2 \frac{3}{4}\right)^{1/2}}$$

$$= \frac{\mu_0 I}{\pi s} \frac{1}{\sqrt{3}} \frac{1}{(1+3)^{1/2}}$$

$$B_{1\,side} = \frac{\mu_0 I}{2\pi\sqrt{3}\,s}$$

and you can do the same for the other 5 sides

$dir(B_{1side}) = \otimes$ into paper (as we have drawn the current)

Step 3 : The total B field is then the sum of the field due to the segments

$$\boxed{B_{\substack{at\\center}} = \frac{6}{2\pi\sqrt{3}} \frac{\mu_0 I}{s}}$$

b) $\vec{B}$ is $\perp$ to the plane of the hexagon.

So a particle with $\vec{v} \parallel \vec{B}$ experiences <u>no magnetic force</u>. The particle will continue moving with v_0 forever

c) $\vec{F}_m = \vec{v}_0 \times \vec{B}$

$|\vec{F}_m| = v_0 B_{tot}$, direction is $\uparrow$

The particle will be deflected in a circle if the field were uniform everywhere in the hexagon. However, the magnitude of B increases closer to the wires. → so radius of circle decreases

In fact the trajectory is complicated and would need a numerical calculation

↓

but the particle always stays in the plane of the hexagon