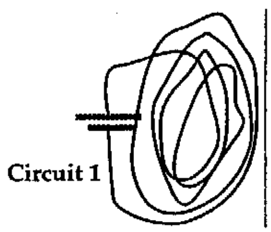
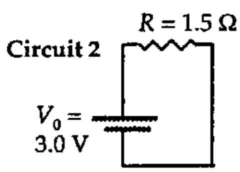
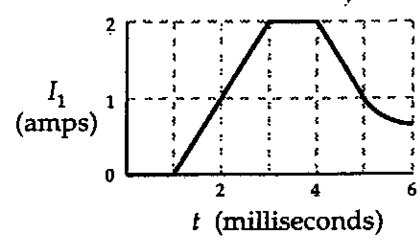


Circuit 1 sits next to circuit 2 on a table. Recall from an earlier problem that "mutual inductance" relates the rate of change in current in one system to the EMF induced by that changing current in *another* system:

$$|\mathcal{E}_{2\text{ induced}}| = M\frac{dI_1}{dt},$$



where I_1 denotes the current in system 1, and $\mathcal{E}_{2\text{ induced}}$ denotes the induced voltage in system 2. Here, the mutual inductance between circuit 1 and circuit 2 is $M = 1.0 \times 10^{-4}$ henrys. When I_1 increases, the induced EMF in circuit 2 is clockwise. Circuit 2 has negligible self-inductance



By fiddling with the power supply on circuit 1, I make I_1 fluctuate as graphed here. Notice that the time axis is subdivided into milliseconds.

The battery and resistor in circuit 2 have voltage $V_0 = 3.0$ V and $R = 1.5\ \Omega$.

(a) Without using formulas, explain why changing the current in circuit 1 induces an EMF in circuit 2.

(b) (*Very hard*) Sketch a rough graph of the current in circuit 2 as a function of time, between $t = 0$ and $t = 6$ milliseconds. Your graph need not be numerically accurate.

(c) What is the current in circuit 2 at time $t = 2$ milliseconds?

(d) What is the current in circuit 2 at time $t = 3.5$ milliseconds?

(e) At time $t = 4.5$ milliseconds?

(f) If circuit 2 were moved farther away from circuit 1, would the mutual inductance go up, go down, or stay the same? Explain your answer.

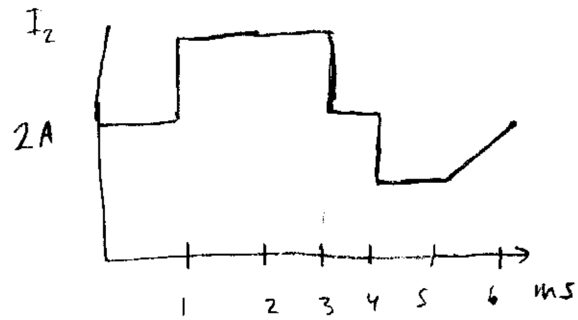
a) changing current in 1 ⇒ changing magnetic flux through 2 ⇒ by Faraday's Law, an EMF is induced in 2

b) $\text{Emf}_{\text{in } 2} \propto \dfrac{d\Phi_{B \text{ in loop } 2}}{dt}$, but $\Phi_{B_2} \propto I_1$

So $\text{EMF}_{\text{in } 2} \propto \dfrac{dI_1}{dt}$ → slope of I_1 vs t graph

The "baseline" current in loop 2 is

$$I_0 = \frac{3.0}{1.5} = 2\text{ A}, \text{ CW}$$



Between $t = 1$ and $t = 3$ ms, I_1 increases CW, Φ_{B_1} increases $\otimes$ dir

so $\Phi_{B_{2,1}}$ increases in $\odot$ dir

flux through 2

$I_{2 ind}$ creates a field to decrease in $\odot$ dir,

so to increase in $\otimes$ dir, so $\underline{\underline{CW}}$

This increases I_2. $\mathcal{E}_{ind}$ = <u>constant</u>, since slope of I_1 vs t is const

Now from $t = 3$ to $t = 4$ ms, no change,

so $I_{ind_2} = 0$ and $I_2 = 2A$

Between $t = 4$ and $t = 5$ ms, steady decrease in I_1 $\Rightarrow$ $I_{2 ind}$ = const, and

opposite in direction, so subtracts from 2A

Between $t = 5$ and $t = 6$ ms,

I_1 decreases, with <u>decreasing slope</u>

(slope becoming less negative)

EMF$_{ind}$ and $I_{2 ind}$ therefore become less negative

c) $t = 2$ ms

$$|\mathcal{E}_{2\,ind}| = M\frac{dI_1}{dt} = (1\times10^{-4}H)\left(\frac{2-0}{0.003-0.001}\right)$$

$$= 0.10\,V$$

This adds to $\mathcal{E}_0 = 3V$

$$\Rightarrow I_2 = \frac{3.1\,V}{1.5\,\Omega} = \underline{\underline{2.07\,A}}$$

d) $t = 3.5$ ms No change $\Rightarrow I_2 = 2A$

e) $t = 4.5$ ms

$$\mathcal{E}_{2\,ind} = -0.10\,V$$

$$I_2 = \frac{2.9\,V}{1.5\,\Omega} = \underline{\underline{1.93\,A}}$$

f) Mutual inductance would <u>decrease</u>;

M expresses how sensitive a configuration's induced EMF is to changes in current.

Farther $\Rightarrow$ less flux $\Rightarrow \frac{d\Phi_B}{dt}$ smaller $\Rightarrow$ smaller M