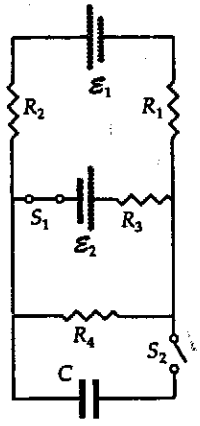
In this circuit, S_1 and S_2 are switches. Current can flow through a switch only when it's "closed." I apologize for this confusing terminology.

The batteries have voltages $\mathcal{E}_1 = 2.0$ V and $\mathcal{E}_2 = 1.5$ V. Neither battery has internal resistance. For the resistors, $R_1 = 3.0\ \Omega$, $R_2 = 5.0\ \Omega$, and $R_3 = 0.50\ \Omega$. The resistance R_4 is unknown. The capacitor has capacitance $C = 3.0 \times 10^{-6}$ coulomb per volt, and is initially uncharged.



When S_1 is closed and S_2 is open, as drawn here, the current through R_4 is measured to be 0.10 amps, leftward.

(a) Solve for R_4. Hint: As far as current is concerned, a "dead end" in the circuit might as well not be there at all. Use this insight to simplify the circuit.

(b) Now S_1 is also opened, so that current flows through neither switch. What's the new current through R_4? Explain intuitively why it changed direction.

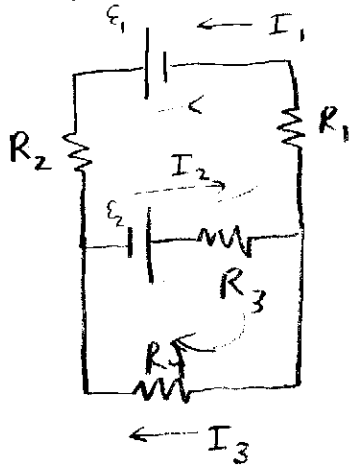
(c) [*Skip parts (c) through (e) if you don't need to deal with capacitors in circuits*] Now, with S_1 still left open, S_2 is closed at time $t = 0$. Immediately after $t = 0$, what's the current through the capacitor? Hint: the capacitor hasn't had time to get charged up by more than a few electron's worth of charge.

(d) *(Very hard)* What's the current through R_4 at time $t = 5$ seconds? Hint: Is the capacitor still getting charged up?

(e) At $t = 5$ s, what's the charge on the capacitor?

a) If S_2 open ignore bottom branch.

Circuit is equivalent to:



Step 1: loops

Step 2: Currents

Step 3: Junction Rule $I_1 + I_3 = I_2$ (1)

Step 4: Loop Rule top loop

$$\mathcal{E}_1 - I_1 R_2 + \mathcal{E}_2 - I_2 R_3 - I_1 R_1 = 0 \quad (2)$$

Bottom Loop: $\mathcal{E}_2 - I_2 R_3 - I_3 R_4 = 0$ ③

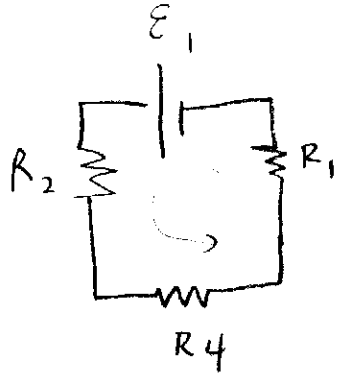
Given $I_3 = 0.1\,A$

have 3 eqns, 3 unknowns

algebra ↳ Get. $R_4 = 12\,\Omega$

b) If S_1 is open, no current can flow through the branch with $\mathcal{E}_2, R_3$

The circuit becomes equivalent to:



Now it's a simple 1-loop circuit

$$R_{eq} = R_1 + R_2 + R_3$$

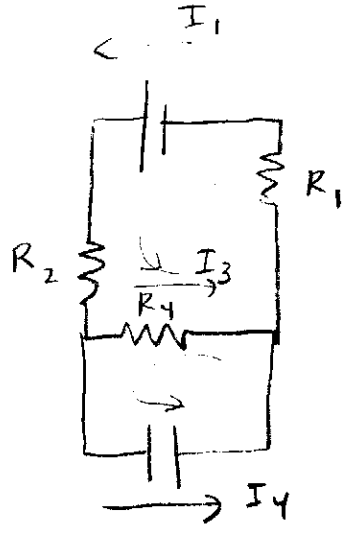
$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{2}{3+5+12} = 0.1\,A$$

Current goes CCW, the way the $\mathcal{E}_1$ battery wants it to go. Before, there was a competition between 2 batteries.

c) S_2 closed this now becomes an

RC charging problem

Equivalent circuit is:



At $t = 0$, there's no charge piled up on the capacitor yet, so current flows easily. Treat it with Kirchoff's Rules (I'm relabeling currents – we're looking for I_4)

$$I_1 = I_4 + I_3$$

Top loop $\quad \mathcal{E}_1 - I_1 R_2 - I_3 R_4 - I_1 R_1 = 0$

Loop with capacitor: $\quad -I_3 R_4 = 0$

$\Delta V_{cap} = 0$ at $t = 0$; against current

so $I_3 = 0 \Rightarrow$ no current through R_4

∴ because capacitor is like a short!

$\Rightarrow$ Now we have

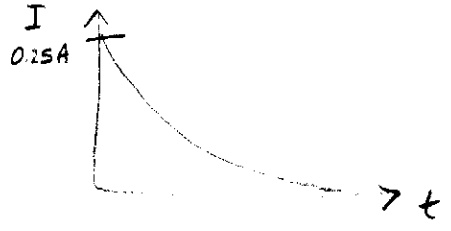
$$I_1 = I_4$$

$$\mathcal{E}_1 - I_1 R_2 - I_1 R_1 = 0$$

} 2 eqns, 2 unknowns

Get $I_4 = \dfrac{\mathcal{E}_1}{R_1 + R_2} = \dfrac{2V}{8\Omega} = \underline{0.25A}$ at $t = 0$

with time, it will look like:



d) We don't really have an equivalent resistance to put into $\tau = RC$, since it's a multi-loop, but order of magnitude: $\tau = RC \sim 5(3\times10^{-6}) \sim 15\times10^{-6} \sim 10^{-5}$ seconds

↓

This is very short compared to 5 seconds

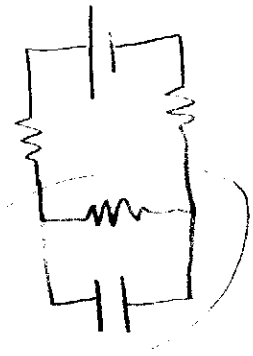
So by 5 seconds the capacitor will be fully charged

When the capacitor is fully charged, there's no current in the capacitor branch

The circuit is equivalent to part b

$\underline{I = 0.1A}$ through R_4 as in part b

e) $C = \frac{Q}{V}$



The loop rule holds for this loop,

$$\Sigma \Delta V_i = 0$$

$$\Delta V_{cap} = \Delta V_{R_4} = I_4 R_4$$

$$= (0.1A)(12)$$

$$\Rightarrow Q = (0.1)(12)(3 \times 10^{-6})$$

$$\boxed{Q = 3.6 \times 10^{-6} C}$$