

I. Induced currents

A. A copper wire loop is placed in a uniform magnetic field as shown. Determine whether there would be a current through the wire of the loop in each case below. Explain your answer in terms of magnetic forces exerted on the charges in the wire of the loop.

- The loop is stationary.

$$\vec{v} = 0 \Rightarrow \vec{F}_B = 0$$

$$\Rightarrow I_{ind} = 0$$

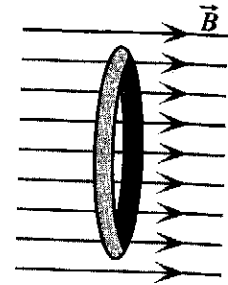
- The loop is moving to the right.

$$\vec{v} \times \vec{B} = 0 \text{ since}$$

$$\vec{v} \text{ tangent to loop } \perp \vec{B}$$

- The loop is moving to the left.

$$\Rightarrow \vec{F}_B = 0 \Rightarrow I_{ind} = 0$$



B. Suppose that the loop is now placed in the magnetic field of a solenoid as shown.

1. Determine whether there would be a current through the wire of the loop in each case below. If so, give the direction of the current. Explain in terms of magnetic forces exerted on the charges in the wire of the loop.

- The loop is stationary.

$$\vec{v} = 0 \Rightarrow \vec{F}_B = 0 \Rightarrow I_{ind} = 0$$

- The loop is moving toward the solenoid.

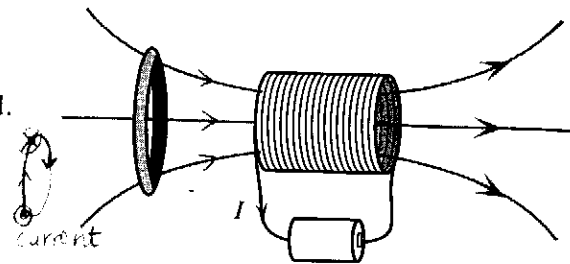
At top of loop: \vec{v}_{loop} \vec{F}_B is into plane

At bottom: \vec{v}_{loop} \vec{F}_B is out of plane

- The loop is moving away from the solenoid.

Top of loop: \vec{v}_{loop} \vec{F}_B out

Bottom of loop: \vec{v}_{loop} \vec{F}_B in



current direction is opposite

2. For each case above in which there is an induced current, determine:

- the direction of the magnetic moment of the loop. (Hint: Find the direction of the magnetic field at the center of the loop due to the induced current in the loop. The magnetic moment is a vector that points in this same direction.)

Moving toward loop: \vec{B}_{ind} to left by RHR

Moving away: \vec{B}_{ind} to right

- whether the loop is attracted toward or repelled from the solenoid.

Moving toward: \vec{B}_{sol} (S-N) repelled

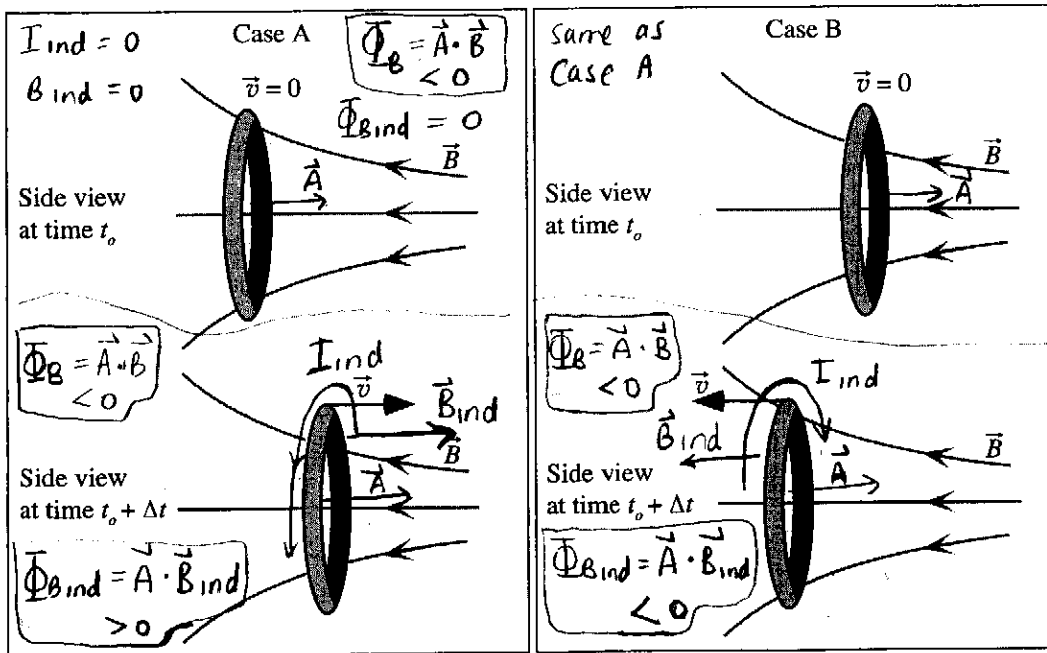
Moving away: \vec{B}_{sol} (S-N) attracted

- whether the force exerted on the loop tends to increase or to decrease the relative motion of the loop and solenoid.

Tries to decrease the relative motion in both cases

C. In each of the diagrams below, the position of a loop is shown at two times, t_0 and $t_0 + \Delta t$. The loop starts from rest in each case and is displaced to the right in Case A and to the left in Case B. On the diagrams indicate:

- the direction of the induced current through the wire of the loop,
- the magnetic moment of the loop,
- an area vector for each loop, \rightarrow choose \mathcal{A}
- the sign of the flux due to the external magnetic field (at both instants), and
- the sign of the induced flux (at both instants).



D. State whether you agree or disagree with each of the students below. If you agree, explain why. If you disagree, cite a specific case for which the student's statement does not give the correct answer. (Hint: Consider cases A and B above.)

Student 1: "The magnetic field due to the loop always opposes the external magnetic field."

No, consider case B at $t_0 + \Delta t$
 \vec{B} and \vec{B}_{ind} are in the same direction

Student 2: "The flux due to the loop always has the opposite sign as the flux due to the external magnetic field."

No, consider case B at $t_0 + \Delta t$
 Φ_B and $\Phi_{B_{ind}}$ are both negative

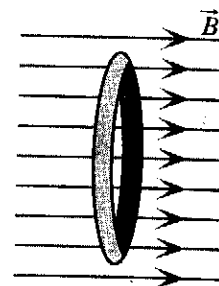
Student 3: "The flux due to the loop always opposes the change in the flux due to the external magnetic field."

Yes, In case A, flux is increasing, induced flux tries to decrease. In case B, flux is decreasing, induced flux tries to increase it.

⇒ Before continuing, check your answers to parts C and D with a tutorial instructor.

II. Lenz' law

A. The diagram at right shows a stationary copper wire loop in a uniform magnetic field. The magnitude of the field is *decreasing* with time.



1. Would you predict that there would be a current through the loop:

- if you were to use the idea that there is a magnetic force exerted on a charge moving in a magnetic field? Explain your reasoning.

No, no current predicted since $\vec{v} = 0$

- if you were to use the reasoning of the student in part D of section I with whom you agreed? Explain.

Yes, since I_{ind} tries to oppose change, so it tries to increase flux

2. It is *observed* that there is an induced current through the wire loop in this case. Use the appropriate reasoning above to find the direction of the current through the wire of the loop.

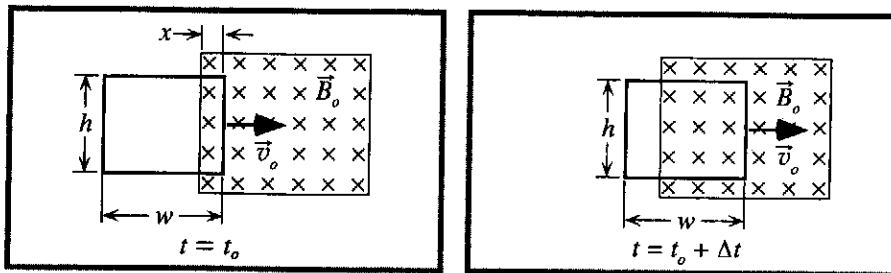
To increase flux, I_{ind} must be:



To understand the interaction between the wire loops and solenoids in section I, we can use the idea that a force is exerted on a charged particle moving in a magnetic field. In each of those cases there was an induced current when there was relative motion between the solenoid and the wire loop. In other situations such as the one above, however, there is an induced current in the wire loop even though there is no relative motion between the wire loop and the solenoid. There is a general rule called *Lenz' law* that we can use in *all* cases to predict the direction of the induced current.

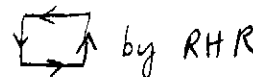
B. Discuss the statement of Lenz' law in your textbook with your partners. Make sure you understand how it is related to the statement by the student with whom you agreed in part D of section I.

C. A wire loop moves from a region with no magnetic field into a region with a uniform magnetic field pointing into the page.

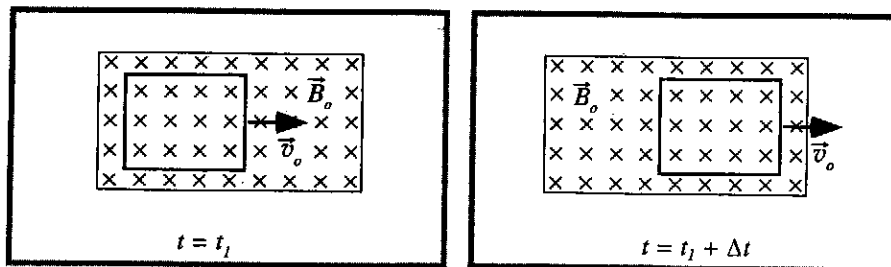


The loop is shown at two instants in time, $t = t_0$ and $t = t_0 + \Delta t$.

1. Is the magnetic flux through the loop due to the external field *positive, negative, or zero*:
 assuming $\vec{A} \parallel \vec{B}_0$
 - a. at $t = t_0$? *positive*
 - b. at $t = t_0 + \Delta t$? *positive*
2. Is the *change* in flux due to the external field in the interval Δt *positive, negative, or zero*?
positive (increase in flux)
3. Use Lenz' law to determine whether the flux due to the induced current in the loop is *positive, negative, or zero*.
Induced current tries to decrease flux (oppose increase) $\Rightarrow \Phi_{ind}$ is negative (\odot)
4. What is the direction of the current in the loop during this time interval?



D. At two later instants, $t = t_1$ and $t = t_1 + \Delta t$, the loop is located as shown.



1. Use Lenz' law to determine whether the flux due to the current induced in the loop is *positive, negative, or zero*. Explain.

Zero. No change in flux

2. Describe the current in the loop during this time interval.

Zero

3. Consider the following student dialogue:

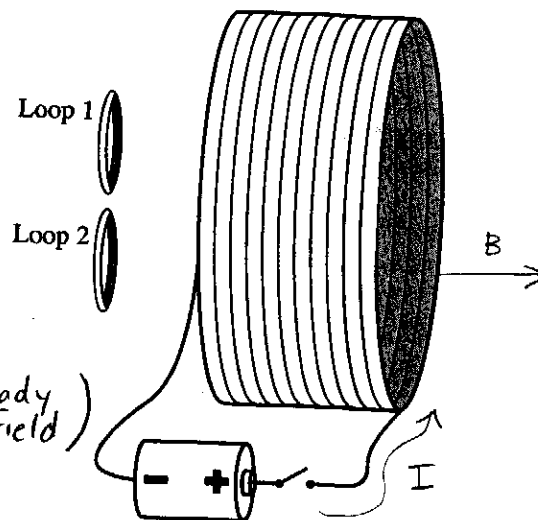
Student 1: "The sign of the flux is the same as it was in part C. So the current here will also be counter-clockwise." *Disagree. I_{ind} relates to change in flux, not flux.*

Student 2: "I agree. If I think about the force on a positive charge on the leading edge of the loop, it points towards the top of the page. That's consistent with a counter-clockwise current." *Disagree. Note that this force is in the opposite direction on the trailing edge \Rightarrow no I_{ind}*

Do you agree with either student? Explain.

I. Faraday's law

Two loops of the same radius are held near a solenoid. Both loops are the same distance from the end of the solenoid and are the same distance from the axis of the solenoid.



A. The resistance of loop 2 is greater than that of loop 1. (The loops are made from different materials.)

1. Is there a current induced through the wire of either of the loops:

- before the switch is closed? Explain.

$$I_{ind} = 0 \quad \text{since} \quad d\Phi_B/dt = 0 \quad (\text{steady field})$$

- just after the switch is closed? Explain.

Yes, because Φ_B is changed

- a long time after the switch is closed? Explain.

No, because Φ_B is steady again.

2. For the period of time that there is a current induced through the wire of the loops, find the direction of the current.

B is to right, and while it's increasing,

I_{ind} tries to decrease it $\Rightarrow \vec{B}_{ind}$ to left $\Rightarrow I_{ind}$ \uparrow

3. The ratio of the induced currents for the two loops is found by experiment to be equal to the inverse of the ratio of the resistances of the loops.

What does this observation imply about the ratio of the induced *emf* in loop 1 to the induced *emf* in loop 2?

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \quad \mathcal{E} = IR \Rightarrow \frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{I_1 R_1}{I_2 R_2} = 1 \Rightarrow \mathcal{E}_1 = \mathcal{E}_2$$

(independent of resistance)

B. Suppose that loop 2 were replaced by a wooden loop.

- Would there still be an *emf* in the loop? Yes. It does not depend on the resistance

- Would there still be a current induced in the wood loop? No. Current cannot flow in an insulator

C. Suppose that loop 2 were removed completely. Consider the circular path that the wire of loop 2 used to occupy.

- Would there still be an *emf* along the path? Explain. Yes; it's independent of matter occupying the space.

- Would there still be a current along the path? Explain. No - for current to exist there must be movement of real charges

The results of the previous exercises are consistent with the idea that a change in the magnetic flux through the surface of a loop results in an *emf* in that loop. If there is a conducting path around the loop (e.g., a wire), there will be a current. The *emf* is independent of the material of which the loop is made; the current is not. It is found by experiment that the induced *emf* is proportional to the rate of change of the magnetic flux through the loop. This relationship is called *Faraday's law*. The direction of any induced current is given by Lenz' law.

- D. Three loops, all made of the same type of wire, are placed near the ends of identical solenoids as shown. The solenoids are connected in series. Assume that the magnetic field near the end of each of the solenoids is uniform.

Loop 2 consists of two turns of a single wire that is twice as long as the wire used to make loop 1. Loop 3 is made of a single wire that is half as long as the wire used to make loop 1.

Just after the switch has been closed, the current through the battery begins to increase. The following questions concern the period of time during which the current is increasing.

1. Let \mathcal{E} represent the induced *emf* of loop 1. Find the induced *emf* in each of the other loops in terms of \mathcal{E} . Explain your reasoning.

$\mathcal{E}_2 = 2\mathcal{E}_1$ because $|\mathcal{E}| = \left| \frac{d\Phi}{dt} \right|$

$\Phi_2 = 2\Phi_1$ because twice the lines thread it

$\Phi_3 = \frac{\Phi_1}{4}$ because area is 4 times smaller, so $\mathcal{E}_3 = \mathcal{E}_1/4$

2. Let R represent the resistance of loop 1. Find the resistance of each of the other loops in terms of R . Explain.

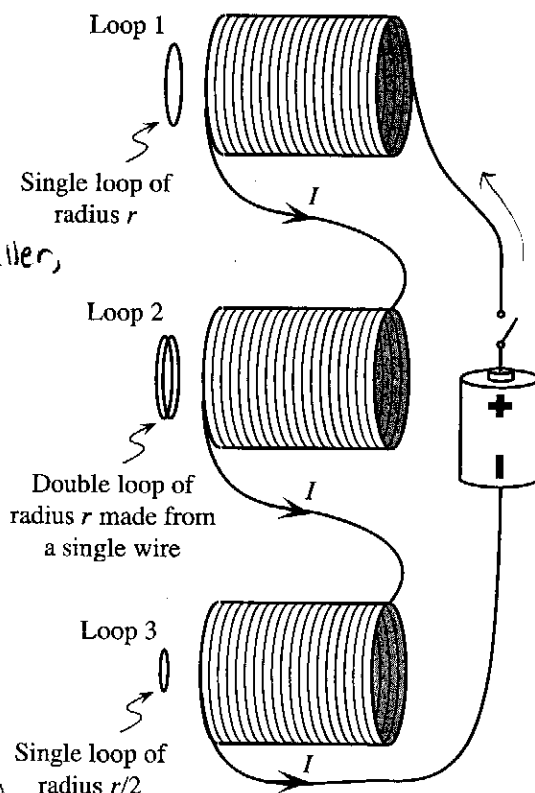
$R_2 = 2R_1$ since $R \propto l$

$R_3 = R_1/2$

3. Find the current induced through the wire of each of the loops in terms of \mathcal{E} and R .

$I_1 = \mathcal{E}/R$ $I_2 = \frac{2\mathcal{E}_1}{2R_1} \Rightarrow I_2 = \mathcal{E}/R$

$I_3 = \frac{\mathcal{E}_1/4}{R_1/2} \Rightarrow I_3 = \frac{1}{2} \mathcal{E}/R$



$\mathcal{E}_1 = \mathcal{E}$
 $R_1 = R$