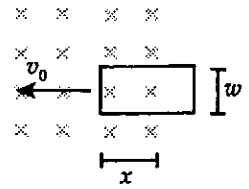


(Hard, but this is a classic exam problem. Come back to it several times before your next test.) Minh holds a rectangular loop of wire, of length  $l$ , width  $w$ , and resistance  $R$ . She pushes it at constant speed  $v_0$  toward a region containing a uniform into-the-page magnetic field,  $B_0$ . As the rectangle enters the magnetic field, Minh maintains its leftward motion at steady speed  $v_0$ .

The following questions refer to the moment at which the rectangle extends distance  $x$  into the magnetic field, as drawn here.



- (a) What is the induced current in the rectangle?
- (b) What magnetic force does the field exert on the rectangle? Give the magnitude and direction.
- (c) What force is Minh exerting on the rectangle? Give the magnitude and direction.
- (d) What is the electrical power dissipated in the rectangle?
- (e) With what power is Minh pushing the rectangle?
- (f) Which, if either, is bigger: Your answer to (d) or your answer to (e)? Explain why, even if you didn't solve parts (d) and (e) completely.

a) Flux inside loop is changing (increasing) in  $\otimes$  dir

$$\Phi_B = B w x$$

$\swarrow$  const     $\swarrow$  const     $\swarrow$  changing

$$|I_{ind}| = \frac{|\mathcal{E}_{ind}|}{R} = \frac{1}{R} \left| \frac{d\Phi_B}{dt} \right| = \frac{B w}{R} \frac{dx}{dt} = \frac{B w v}{R}$$

By Lenz's Rule, this current must create a field which opposes the change  $\Rightarrow$  a field in  $\odot$  dir so it's CCW

b)  $\otimes \downarrow I_{ind}$

Force on current carrying wire in  $\vec{B}$  field is

$$\vec{F} = I \vec{l} \times \vec{B} = \frac{B w v}{R} w B = \frac{B^2 w^2 v}{R}$$

from part a

direction to right

(opposing motion)

c) If speed is constant,  $a = 0$

$$\Sigma F_{\text{tot}} = 0$$

$$F_{\text{Minh}} = F_{\text{mag}}$$

$$\underline{\underline{F_{\text{Minh}} = \frac{B^2 w^2 v}{R}}}, \text{ direction to } \underline{\underline{\text{left}}}$$

$$d) P = I^2 R = \frac{B^2 w^2 v^2}{R^2} R = \underline{\underline{\frac{B^2 w^2 v^2}{R}}}$$

$$e) P_{\text{Minh}} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{x}}{dt} = \vec{F}_{\text{Minh}} \cdot \vec{v} = \frac{B^2 w^2 v}{R} \cdot v = \underline{\underline{\frac{B^2 w^2 v^2}{R}}}$$

f) d&e have the same answer. This is not a coincidence! Minh does work to push.

But this work is not converted into KE (loop does not increase speed)  $\Rightarrow$  it goes to heat. The rate at which energy goes to heat is the same as the power dissipated by the current in the wire!!