36. (II) Consider a straight section of wire of length l, as in Fig. 28–42, which carries a current I. (*a*) Show that the magnetic field at a point P a distance R from the wire along its perpendicular bisector is

$$B = \frac{\mu_0 I}{2\pi R}\frac{l}{(l^2 + 4R^2)^{\frac{1}{2}}}.$$

(*b*) Show that this is consistent with Example 28–9 for an infinite wire.

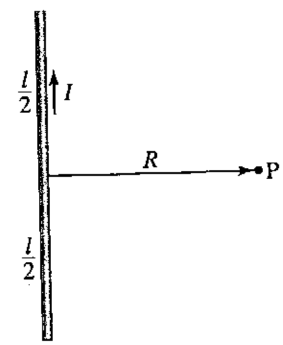
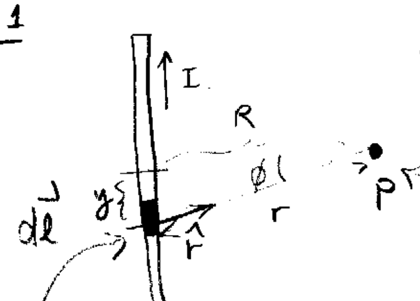


FIGURE 28–42 Problem 36.

a) Step 1



Just a single segment, current I

finding field here

Step 2

- choose $d\vec{l}$, draw $\hat{r}$ from $d\vec{l}$ to P. $d\vec{l}$ is at y
- $dB = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$ } total field is the integral $\int dB$

$$B = \int dB = \int_{y=-l/2}^{l/2} \frac{\mu_0 I}{4\pi} \frac{|d\vec{y} \times \vec{r}|}{r^3}$$ } now change to variables easier to integrate over

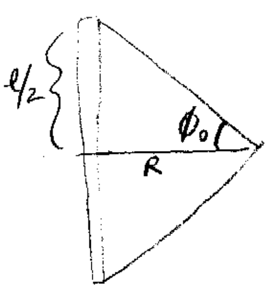
$$d\vec{y} \times \vec{r} = (dy)(r)\sin(\phi + \pi/2) = r\,dy\cos\phi$$

$$\tan\phi = \frac{y}{R}, \quad \cos\phi = \frac{R}{r} \Rightarrow r = \frac{R}{\cos\phi}$$

$$y = R\tan\phi$$

$$dy = R\sec^2\phi\, d\phi = \frac{R}{\cos^2\phi}d\phi$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi}\int_{-\phi_0}^{\phi_0} \frac{(R/\cos^2\phi)\, d\phi \cos\phi}{(R/\cos^2\phi)^2} = \frac{\mu_0 I}{4\pi R}(\sin\phi)\Big|_{-\phi_0}^{\phi_0}$$



$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} 2\sin\phi_0$$

$$\sin\phi_0 = \frac{l/2}{((l/2)^2 + R^2)^{1/2}} = \frac{l}{(l^2+4R^2)^{1/2}}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi R}\frac{l}{(l^2+4R^2)^{1/2}}}$$

Step 3: only a single segment to add up

Direction given by RHR (into page at P)

b) Infinite wire: $l \to \infty$ $\quad B = \frac{\mu_0 I}{2\pi R}\frac{l}{l(1+\underbrace{4R^2/l^2}_{\to 0})^{1/2}}$

$B \sim \frac{\mu_0 I}{2\pi R}$, same as for infinite wire