37. (II) A segment of wire of length l carries a current I as shown in Fig. 28–43. (a) Show that for points along the positive x axis (the axis of the wire), such as point Q, the magnetic field **B** is zero. (b) Determine a formula for the field at points along the y axis, such as point P.

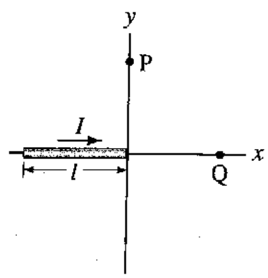
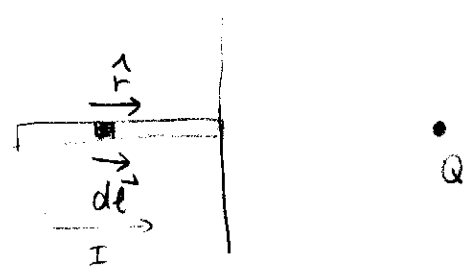


FIGURE 28–43
Problem 37.

a) <u>Step 1</u> : again, just a single segment



<u>Step 2</u> Choose $d\vec{l}$

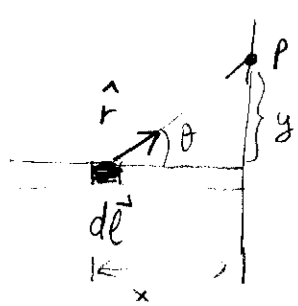
draw $\hat{r}$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$: but here $d\vec{l} \parallel \hat{r}$

$\Rightarrow d\vec{l} \times \hat{r} = 0$

$\Rightarrow$ $\boxed{\vec{B} = 0 \text{ along axis of wire}}$

b)



Same $d\vec{l}$, but now

$\hat{r}$ points to P

Define x, y, θ

$d\vec{l} = d\vec{x}$

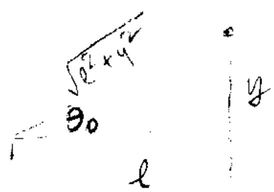
$$\tan\theta = \frac{y}{x} \quad , \quad \sin\theta = \frac{y}{r}$$

$$B = \int dB = \int \frac{\mu_0 I}{4\pi} \frac{|d\vec{x} \times \vec{r}|}{r^3}$$

$$d\vec{x} \times \vec{r} = r\,dx \sin\theta \quad , \quad \mathrm{dir}(d\vec{x} \times \hat{r}) = \odot$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-l}^{0} dx \frac{r \sin\theta}{r^3}$$

Now change variables to integrate over θ



$$x = \frac{y}{\tan\theta}$$

$$dx = \frac{y}{\sin^2\theta}\, d\theta$$

$$r = \frac{y}{\sin\theta}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{\theta_0}^{\pi/2} \frac{\frac{y}{\sin^2\theta}\, d\theta\, \frac{y}{\sin\theta}\sin\theta}{\left(\frac{y}{\sin\theta}\right)^3}$$

$$= \frac{\mu_0 I}{4\pi} \int_{\theta_0}^{\pi/2} \frac{\sin\theta}{y}\, d\theta = \frac{\mu_0 I}{4\pi} \left(\frac{-\cos\theta}{y}\right)\Bigg|_{\theta_0}^{\pi/2}$$

$$= \frac{\mu_0 I}{4\pi y} \cos\theta_0 \quad , \quad \cos\theta_0 = \frac{l}{\sqrt{l^2+y^2}}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I l}{4\pi y} \frac{1}{(l^2+y^2)^{1/2}}}$$