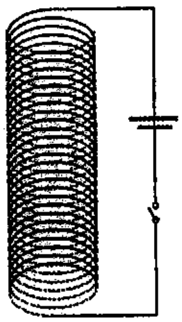


Consider this cylindrical solenoid, consisting of N coils (turns). It has radius r and length D, where $D \gg r$. Wires connect the ends of the solenoid to a battery. But the circuit isn't complete until the switch is closed.

(a) Suppose the battery has voltage V and the solenoid has total resistance R. Ohm's law implies that the current through the solenoid will reach $I_{final} = V/R$ after we close the switch.

But according to actual measurements, when I close the switch, the current does not *immediately* shoot up to I_{final}. The current takes a few tenths of a second to reach I_{final}. By contrast, when I connect a regular resistor to a battery, the current shoots up to I_{final} almost immediately. *Explain in words why the current in the solenoid takes significant time to reach its final value.* Check your answer against mine before proceeding to part (b), because this verbal explanation motivates everything that follows.

(b) As a first step to finding the solenoid's inductance, suppose that arbitrary current I flows through the solenoid. What is the total magnetic flux through *all coils* of the solenoid? Hint: Use Ampere's law to find the magnetic field inside the solenoid.

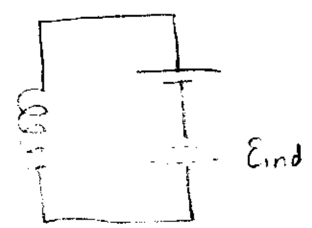
(c) Suppose the current through the solenoid starts increasing, at known rate dI/dt. What is the corresponding rate of change of the magnetic flux through the solenoid?

(d) What is the induced voltage in the solenoid coils? Does this induced voltage "help" or "hinder" the increasing current?

(e) The inductance L is defined by $\mathcal{E}_{induced} = -L(dI/dt)$, where $\mathcal{E}_{induced}$ denotes the voltage induced by the changing current. Explain in physical terms what inductance means. What is the inductance of this solenoid?

a) After switch closed

$I \uparrow \Rightarrow \Phi_B \uparrow \Rightarrow \mathcal{E}_{ind}$ opposite battery's emf



The induced emf "fights" the battery, so delays the increase of current to I_{final}

b) Inside solenoid $\Phi_B = N \Phi_{B\,1\,coil}$

$B = \mu_0 n I$, where $n = \frac{N}{D}$

$$\Phi_B = N\left(\mu_0 \frac{N}{D} I \pi r^2\right)$$

$$\boxed{\Phi_B = \frac{\mu_0 N^2}{D} \pi r^2 I}$$

(Note $\Phi_B \propto I$)

c) $$\frac{d\Phi_B}{dt} = \underbrace{\frac{\mu_0 N^2}{D} \pi r^2}_{\text{const}} \frac{dI}{dt}$$

d) $$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 N^2}{D} \pi r^2 \frac{dI}{dt}$$

hinders

compare

e) $$\mathcal{E}_{ind} = -L\frac{dI}{dt}$$

self-inductance of solenoid

$$\boxed{L = \frac{\mu_0 N^2}{D} \pi r^2}$$

High L ⇒ change in current opposed strenuously

Small L ⇒ not very sensitive to change in current