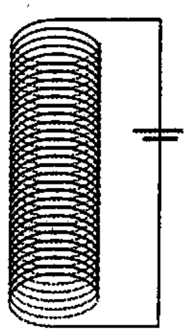


Consider a solenoid consisting of N coils, each with resistance R_0. (A solenoid is essentially an inflexible slinky.) The solenoid is hooked up to a battery of voltage V_0, as drawn here. The solenoid has length D and radius s, where $s \ll D$.

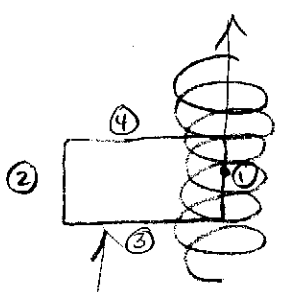
Throughout this problem, neglect the magnetic fields generated by the straight wires connecting the solenoid to the battery. Also, derive any solenoid-specific formulas from basic laws. Don't just plug in pre-derived solenoid formulas from your textbook.

(a) To good approximation, what is the magnetic field strength on the central axis of the solenoid, midway between the two ends of the solenoid?

(b) To good approximation, what is the magnetic field strength a distance $s/2$ from the central axis of the solenoid, midway between the two ends of the solenoid?

(c) Suppose you shoot a particle of charge q, mass m, and velocity v_0 down the central axis of the solenoid, from the top end of the solenoid towards the bottom end. This happens in outer space, where we can neglect gravity. Does the particle come out the other end? If not, why not? If so, at what speed?

a)



Want magnetic field in center

Here, use Ampère's Law $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$

B field is along the axis (RHR)

choose this loop. Why is this a good one?

Let me count the ways...

- it encloses some current
- inside $d\vec{\ell} \parallel \vec{B}$, so $\vec{B}\cdot d\vec{\ell} = B\,d\ell$, and B is const
- on one side $\vec{B} = 0$
- on sides 3 & 4 $d\vec{\ell} \perp \vec{B}$ inside, $\vec{B} = 0$ outside
  $\Rightarrow d\vec{\ell}\cdot\vec{B} = 0$

LHS of Ampère's Law:

because $\vec{B} = 0$ because $\vec{B} \perp d\vec{\ell}$

$$\underset{\substack{\text{closed}\\\text{loop}}}{\oint} \vec{B}\cdot d\vec{\ell} = \int_{1} \vec{B}\cdot d\vec{\ell} + \int_{2} \vec{B}\cdot d\vec{\ell} + \int_{3} \vec{B}\cdot d\vec{\ell} + \int_{4} \vec{B}\cdot d\vec{\ell}$$

$$= B \int d\ell$$

LHS $= B\cdot \ell$, where ℓ is the length of the loop inside the solenoid

$$RHS = \mu_0 \underbrace{I_{encl}}_{\text{what is this?}}$$

Suppose we have n turns per unit length

$$n = \frac{N}{D}$$

The no. of turns enclosed in the loop is then $l \frac{N}{D}$

This is the no. of times the current I pierces the loop. So $I_{encl} = \frac{lN}{D} I$

Now reunite LHS with RHS

$$B \cdot \cancel{l} = \mu_0 \frac{\cancel{l} N}{D} I$$

$R_{tot} = NR_0$

$$\boxed{B = \mu_0 \frac{N}{D} I}$$

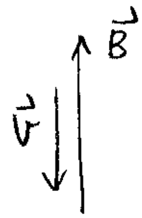
In terms of given quantities $I = \frac{V_0}{NR_0} \Rightarrow \boxed{B = \frac{\mu_0 V_0}{D R_0}}$

Notice that we can make exactly the same argument no matter where we put the loop $\Rightarrow$ B is constant everywhere inside the solenoid

b) Since $\vec{B}$ = constant everywhere

$$\boxed{B = \frac{\mu_0 V_0}{D R_0}}$$ at a point off-axis

c)



$$\vec{F}_m = q\vec{v} \times \vec{B}$$

$\vec{v}$ anti $\parallel$ $\vec{B} \Rightarrow \vec{F}_m = 0$

<u>Particle passes through undeflected</u>