Alternating Current Equation Summary

This is not so much a How-To as a summary of what the main AC equations *mean* in words, to help you figure out where to apply them. Probably the easiest way to think of AC circuits is by analogy to DC circuits, in the framework of "Ohm's Law for AC circuits".

1. $$V = V_0 \sin \omega t$$

   This is the power supply EMF for an AC supply. The voltage is *forced* to oscillate in time like this. V_0 is the peak voltage, a.k.a. the maximum voltage. The frequency of the voltage wiggle is ω.

2. $$I = I_0 \sin(\omega t - \phi)$$

   This is the AC current. The current in an AC circuit wiggles with the same ω as the power supply voltage, but it's *out of phase*, i.e. shifted (leading or following) with respect to $V(t)$, by a phase ϕ.

3. $$I_0 = V_0/Z$$

   This is "Ohm's Law for AC circuits". It tells you what the peak current is for a particular power supply peak voltage. Z is the **impedance**, which plays the role of resistance for AC circuits.

   For rms voltages and currents (which are just related to the peak voltages and currents by $1/\sqrt{2}$), you can write this as

   $$I_{\text{rms}} = V_{\text{rms}}/Z.$$

4.

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

This is the value of impedance. It tells you how much the circuit elements, L,C, and R, impede the flow of current. The impedance depends on the values of L,C, and R, and also on the frequency.

5.

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}$$

This is the phase shift from equation 2. It tells you how much the current lags or leads the voltage, depending on what the frequency is and what the circuit element values are. We can also write: $\cos\phi = R/Z$, $\sin\phi = \frac{\omega L - 1/\omega C}{Z}$.

6. At **resonance**, the current has its largest amplitude (as a function of frequency). This happens when $\omega = 1/\sqrt{LC}$. The phase shift is also zero at resonance.

With these equations you can solve most AC problems. So where all do these equations come from? They come from solving the differential equation that comes from writing Kirchoff's loop equation around the RLC loop, and assuming the sinusoidal power supply voltage (see the RLC How-To. (The actual solution is neither short nor cute, and you won't be expected to reproduce it in detail; you should just understand the general idea of where they came from, and the results.)