How To Find the Magnetic Field for a Current Distribution Using Ampère's Law

Ampère's Law is the analog to Gauss' Law for magnetic fields (with B field analogous to E and I analogous to Q), and you can approach problems in a rather similar way to those in the Gauss' Law howto.

The main difference for Ampère's Law is that you choose a *2-dimensional loop* rather than a 3-dimensional surface to integrate over in the LHS.

1. First, draw a picture. It often helps to draw the magnetic field lines.

2. Choose an Ampèrian loop in the region where you want to find the field. (Note that a "loop" need not be curved. It could be a rectangle with straight sides. It just has to be a closed path). Generally you want to pick a loop which will make the integral easy. For instance, if you pick a path (or part of a path) where $\vec{B} = 0$ then $\vec{B} \cdot d\vec{l}$ will be zero over that path (or part of it). If the path is *perpendicular* to the direction of $\vec{B}$, $\vec{B} \cdot d\vec{l}$ will also be zero. If you choose a path where $\vec{B}$ is *parallel* to $d\vec{l}$, and *constant* over the path, then $\vec{B} \cdot d\vec{l}$ will just be $B \oint dl$: you can pull the constant B out of the integral. For instance, B is constant a fixed distance away from a wire, so a loop concentric with the wire will work well.

3. Apply Ampère's Law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$. Evaluate the LHS line integral, which should be straightforward if you have wisely chosen your loop.

4. Evaluate the RHS of Ampère's Law, $\mu_0 I_{\text{encl}}$. I_{encl} is the current *flowing through* the loop you have chosen. If the current density is not constant, you may have to set up an integral to add up all the current inside your loop. To do this:

   - $I_{\text{encl}} = \int dI = \int J dA$
   - Pick a surface element dA, which has the same symmetry as the current distribution.
   - Plug in the current density J, and integrate over the surface inside your Ampèrian loop.

   Also, remember that currents flowing in opposite directions can be considered to have opposite signs.

5. Set LHS of Ampère's Law equal to RHS, and solve for B.

Notice that you can also do the inverse thing and use Ampère's Law to find current inside a loop for a given $\vec{B}$!