How To Find the Electric Field for a Continuous Distribution of Charges

For a continuous distribution of charge, it's really the same thing as for point charges, except that you treat the continuous distribution as if it is a bunch of infinitesimally small point charges added together. So the total field at X, which is the sum of fields due to each piece Δq_i, $E_{tot} = \sum_{i=1}^{n} \frac{k\Delta q_i}{r_i^2}$, becomes an integral $E_{tot} = \int \frac{kdq}{r^2}$. To actually calculate this integral, the trick is to convert dq into some quantity you can integrate over.

1. Identify the spot X where you want to find the field.

2. Mentally divide the continuous charge distribution up into a bunch of tiny pieces which are equivalent to point charges. Exactly how to do this depends on the shape of the object. For example, if you have a line charge, cut it up into little pieces dq along the line.

3. For each piece dq, draw the vector $d\vec{E}_i$ at X which is the contribution to the electric field due to dq, remembering that direction of $\vec{E}$ is away from positive charges and towards negative ones. When considering the field due to dq, ignore the presence of any other charges.

4. Find the magnitude of this contribution to the field at X: $dE_i = kdq/r_i^2$, where r_i is the distance from the source charge dq to the spot X you're at.

5. Decompose the fields $d\vec{E}$ into components in your chosen coordinate system.

6. Now add the fields $d\vec{E}_i$ vectorially, i.e. add up the $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ components. Note that you can often use symmetry to simplify your answer (for example, for an infinite line charge, components parallel to the line cancel). The sum in a particular direction will be an integral of the type $E_{tot} = \int \frac{kdq}{r^2}$.

7. To evaluate this integral, you need to convert dq into some space coordinate you can integrate over, using what you know about the charge density distribution. For example: if you have a uniform line charge, $dq = \lambda dx$, where λ is the line charge density (charge per unit length).

8. Substitute the relation between charge and space coordinate into the integral, and crank it through to get the total field.

An example of a problem which can be approached in this way is pset #2 problem 2.