

How To Find the Electric Field for a Continuous Charge Distribution Using Gauss' Law

In principle, you don't need Gauss' Law to evaluate the electric field; you can use the previous "How To Find the Electric Field for a Continuous Distribution of Charges" and directly evaluate the field by integrating the contributions from all the little pieces dq . However often you can get some really messy integral so frequently in practice this approach doesn't work too well. But when you have a charge distribution with *symmetry* (like a sphere or cylinder), then the Gauss' Law method comes to the rescue and makes your life easier. Here's how to approach such problems.

1. First, draw the electric field lines, using considerations of symmetry. For instance, they radiate out from a spherically symmetric charge distribution.
2. Decide on a Gaussian surface in the region where you want to find the field. Generally you want to pick one which will make the integral easy, and a surface with the same symmetry as the charge distribution will usually do that. Ask yourself: *over which surface is the magnitude of E going to be constant?* This would be, for instance, a spherical surface for a spherical charge distribution, a cylindrical surface for a cylindrical charge distribution, a plane for an infinite planar charge distribution, etc. You would also like to try to arrange that the surface be oriented parallel or perpendicular to the field, so that $\vec{E} \cdot d\vec{A}$ is either 0 or just $E dA$. This could be true for the whole surface (e.g. for spherical surface $\vec{E} \cdot d\vec{A} = E dA$), or for each piece of the surface (e.g. for a box around an infinite plane, $\vec{E} \cdot d\vec{A} = E dA$ on the sides parallel to the surface, and $\vec{E} \cdot d\vec{A} = 0$ on the sides perpendicular to the surface.)
3. Apply Gauss' Law, $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$. Evaluate the LHS surface integral, which should be straightforward if you have wisely chosen your surface.
4. Evaluate the RHS of Gauss' Law, $\frac{Q_{\text{in}}}{\epsilon_0}$. This is the charge *inside* the surface you have chosen. If the charge density is not constant, you may have to set up an integral to add up all the charge inside your surface. To do this:

- $Q_{\text{in}} = \int dq = \int \rho dV$

- Pick a volume element dV , which has the same symmetry as the charge distribution.
- Plug in the charge density ρ , and integrate over the volume inside your Gaussian surface.

Also, remember that $Q_{\text{in}} = 0$ inside a conductor.

5. Set LHS of Gauss' Law equal to RHS, and solve for E .

Notice that you can also do the inverse thing and use Gauss' law to find charge inside a surface for a given \vec{E} !